Mathematics – Decimals

Year 9

What do I need to be able to do?

Four operations with decimals

Round numbers

to powers of 10

to a given number of decimal places

to a given number of significant figures

Know the difference between **rounding** numbers and **truncating** numbers

Estimate calculations

Understand and use limits of **accuracy**

Understand and use **error interval notation**

Keywords:

Accuracy: the exact value of something

Estimate: make an informed guess using available information

Round: estimate a number to a level of accuracy

Decimal places: the digits after the decimal point

Significant figures: starting with the 1st non-zero digit and decreasing in place value

Truncated: the number cut off at a level of accuracy

Limits: the highest and lowest values a number could be prior to rounding

Round to powers of 10:

Nearest - look at the next place value and decide:

One: 0.0 - 0.4 round down, 0.5 - 0.9 round up

Ten: 0 - 4 round down, 5 - 9 round up

Hundred: 0 - 50 round down, 50- 99 round up

Thousand: 0 - 499 round down, 500 - 999 round up

"zero to 4 let it go, 5 or above give it a shove"

Round to decimal places:

Nearest - look at the next decimal place value:

If it is 0, 1, 2, ,3 or 4 round down.
If it is 5, 6, 7, 8, or 9 round up

Examples:

Round a) 6.**4**7 to 1dp = 6.5 (7 so round up)

b) 4.54**2** to 2dp = 4.54 (2 so round down)

c) 5.99**9** to 2dp = 6.00 (9 so round up)

Round to significant figures:

Nearest - start at the first non-zero value:

If it is 0, 1, 2, ,3 or 4 round down.
If it is 5, 6, 7, 8, or 9 round up

Examples:

Round a) 6.**4**7 to 1 sig fig = 6 (4 so round down)

b) 4.54**2** to 3 sig fig = 4.54 (2 so round down)

c) 5.99**9** to 2 sig fig = 6.0 (9 so round up) - note the extra zero to ensure 2 significant figures

Estimating - round each number to 1 sig fig then do the calculation:

a) 34.3 x 0.52 ≈ 30 x 0.5 = 15 (≈ means approximately)

b) $\frac{231 + 45.9}{0.46} \approx \frac{200 + 50}{0.5} = \frac{250}{0.5} = 500$

Truncating Numbers - using the most significant part of the number and ignoring rounding rules:

Example: Truncate to 1 sig fig

a) 39.654 = 30 (just the tens column)

B) 0.4666 = 0.4

Limits of accuracy:

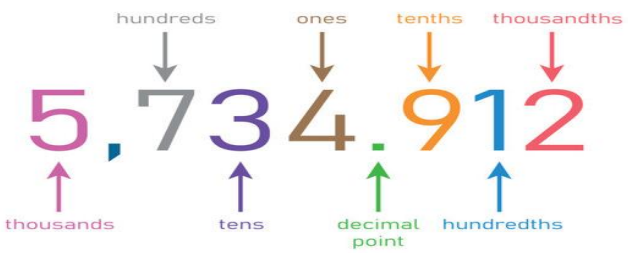
These are called the upper and lower bound.

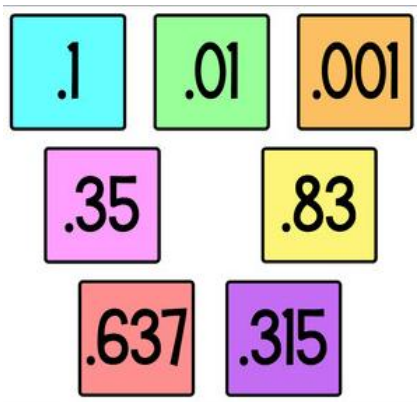
They are always half the given accuracy level each way:

For example, nearest 100 is 50 either way so the lower and upper bounds of 500 are *450 and 550.*

It is written: **450 ≤ 500 < 550**

Lower Bound **Upper Bound**





St Bede's Roman Catholic High School

Year 9 Autumn 2

MATHS

Four Operations with Decimals

Addition and Subtraction

4	.	3	8	
7	.	9	0	+

0 can be used to fill empty places with value

The decimal place acts as the placeholder and aligns the other values

factors

Division with decimals

The placeholder in division methods is essential – the decimal lines up on the dividend and the quotient

24 ÷ 0.02 → 24 ÷ 0.2 → 240 ÷ 2

All give the same solution as represent the same proportion.
Multiply the values in proportion until the divisor becomes an integer

Multiplication methods

Less effective method especially for bigger multiplication

H	T	O
1	8	7
×		9

Long multiplication (column)

×	100	80	7
9			

Grid method

1	8	7	
1	8	7	
1	8	7	
1	8	7	
1	8	7	
1	8	7	
+	1	8	7

Repeated addition

Multiplication with decimals

Perform multiplications as integers
e.g. 0.2 x 0.3 → 2 x 3

Make **adjustments** to your answer to match the question: 0.2 x 10 = 2
0.3 x 10 = 3
Therefore 6 ÷ 100 = 0.06

Estimations: Using estimations allows a 'check' if your answer is reasonable

Division methods

3584 ÷ 7 = 512

Short division

$$7\overline{)\,3\,{}^{3}5\,8\,{}^{1}4}\quad 5\,1\,2$$

Complex division

÷ 24 = ÷ 6 ÷ 4

Break up the divisor using

Higher Tier Topics Only

Solve problems involving upper and lower bounds

Addition (A + B)
Upper Bound is Max A + Max B
Lower Bound is Min A + Min B

Multiplication (A x B)
Upper Bound is Max A x Max B
Lower Bound is Min A x Min B

Subtraction (A + B)
Upper Bound is Max A - Min B (Biggest Gap)
Lower Bound is Min A - Max B (Smallest Gap)

Division (A + B)
Upper Bound is Max A ÷ Min B (Biggest Gap)
Lower Bound is Min A ÷ Max B (Smallest Gap)

Example 1
The dimensions of the rectangle are given to the nearest cm.

Write, the upper and lower bounds of the perimeter of the rectangle:
Upper Bound (18.5 + 7.5) x 2 = 52cm
Lower Bound (17.5 + 6.5) x 2 = 48cm

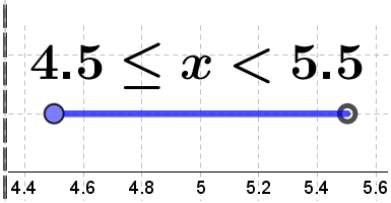
We write this as **48 ≤ Perimeter ≤ 52**

Example 2
Aisha runs 100 m in 15 seconds.

Find the upper and lower bounds of Aisha's speed if the numbers are given to the nearest integer.
Upper: 100.5 ÷ 14.5 = 6.9 m/s (1 d.p.)
Lower: 99.5 ÷ 15.5 = 6.4 m/s (1 d.p.)

What do I need to be able to do?

Solve problems involving upper and lower bounds
Convert a recurring decimal to a fraction



Keywords:

Error Bounds: the points where you change the number being used as an estimate

Error Interval: the range of values between the bounds shown as an inequality

Recurring decimal: repeated pattern of numbers.

Maximum Value: Largest ***possible*** solution

Minimum Value: Lowest ***possible*** solution

Convert a recurring decimal to a fraction

- Convert 0.8 to a fraction
  x = 0.8
  10x = 8.8 (match the recurring decimal)
  10x - x = 8.8—0.8
  9x = 8 (divide 9)
  $x = {}^{8}/_{9}$

- Convert 0.23 to a fraction
  x = 0.23
  10x = 2.3232 (decimal doesn't match)
  100x = 23.23 (match the recurring decimal)
  100x - x = 23.23- 0.23
  99x = 23 (divide 99)
  $x = {}^{23}/_{99}$

- Convert 0.14545 to a fraction
  x = 0.14545
  10x = 1.4545 (decimal doesn't match)
  100x = 14.5454 (decimal still doesn't match)
  1000x = 145.4545 (decimal matches 10x)
  1000x - 10x = 145.45 - 1.45
  990x = 143 (divide 990)
  $x = {}^{143}/_{990}$

What do I need to be able to do?

- Substitute into algebraic expressions
- Simplify algebraic expressions
- Multiply out a single bracket
- Expand multiple single brackets and simplify
- Factorise into a single bracket
- Find the product of two brackets
- **Find the product of three brackets**
- Factorise quadratics of the form $x^2 + bx + c$ into 2 brackets
- **Factorise quadratics of the form $ax^2 + bx + c$**
- **Write a quadratic in the form $(x + a)^2 + b$**

Keywords:

Operation: a mathematical process
Inverse: the operation that undoes what was done by the previous operation. (The opposite operation)
Commutative: the order of the operations do not matter.
Variable: a letter that stands for a number
Substitute: replace one variable with a number or another variable.
Term: number or variable in an expression or equation
Coefficient: number in front of the variable
Like terms: variables which are the same
Constant: a term which is not a variable (a number)
Expression: a maths sentence with a minimum of two terms and at least one operation (no equals sign)
Simplify: write the expression as simply as possible
Evaluate: work out the value of
Expand: multiply out brackets
Factorise: put an expression into brackets
Quadratic: an expression with a square (x^2) term and no terms with higher powers

Substitute into algebraic expressions

If $y = 7$, evaluate $3y - 2$ — $3y$ means 3 lots of $y = y + y + y$

$3y - 2 = 7 + 7 + 7 - 2 = \mathbf{19}$

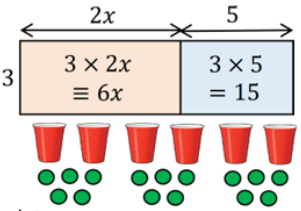
If $a = 3$ and $b = 5$, evaluate $2a^2 + 4b$ — a^2 means $a \times a$; $2a^2$ means $2 \times a \times a$

$2a^2 + 4b = 2 \times 3 \times 3 + 4 \times 5$
$= 18 + 20$
$= \mathbf{38}$

Multiply out a single bracket

Expand $3(2x + 5)$

$2x + 5$	$2x + 5$	$2x + 5$
x x 5	x x 5	x x 5
$6x + 15$		



Expand and simplify where appropriate

$7(3 + a) = 21 + 7a$

$2(5 + a) + 3(2 + a) = 10 + 2a + 6 + 3a$
$= 5a + 16$

Simplify algebraic expressions

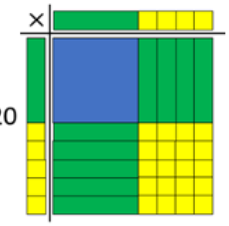
Simplify $2a + 5a + 3a$ (Collect like terms)
$= 10a$

Simplify $a + 3b + 3a^2 - 2b$

Find the product of two brackets

Expand $(x + 5)(x + 4)$

$(x + 5)(x + 4) \equiv x^2 + 5x + 4x + 20$
$\equiv x^2 + 9x + 20$



Find the product of three brackets

$(x + 3)(x + 4)(x + 2)$

$(x + 3)(x + 4) = x^2 + 7x + 12$

$(x^2 + 7x + 12)(x + 2)$

×	x	3
x	x^2	$3x$
4	$4x$	12

×	x^2	$7x$	12
x	x^3	$7x^2$	$12x$
2	$2x^2$	$14x$	24

$(x + 3)(x + 4)(x + 2) = x^3 + 9x^2 + 26x + 24$

Factorise quadratics of the form $ax^2 + bx + c$

$2x^2 - 3x - 2$ (a b c)

$ac = -2 \times 2 = -4$

Look for other products that = -4 which sum (add) to b (= -3)

$-4 \times 1 = -4$

$-4 + 1 = -3 = b$

Split the -3x into -4x + 1x

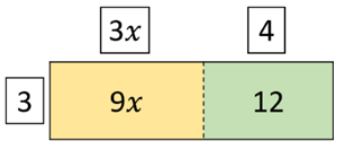
$2x^2 - 4x + x - 2$

$2x(x - 2) + 1(x - 2)$

$(2x + 1)(x - 2)$

Factorise into a single bracket

$9x + 12 \equiv 3 \times 3x + 3 \times 4$



$\equiv 3(3x + 4)$

Write a quadratic in the form $(x + a)^2 + b$

$y = x^2 - 2x - 15$
$y = x^2 - 2x + 1 - 16$
$y = (x - 1)^2 - 16$