

Mathematics – Multiplicative Change

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems and explain direct proportion
- Use conversion graphs to make statements, comparisons and form conclusions.
- Understand and use scale factors for length

Keywords

- Proportion:** a statement that links two ratios
- Variable:** a part that the value can be changed
- Axes:** horizontal and vertical lines that a graph is plotted around
- Approximation:** an estimate for a value
- Scale Factor:** the multiple that increases/ decreases a shape in size
- Currency:** the system of money used in a particular country
- Conversion:** the process of changing one variable to another
- Scale:** the comparison of something drawn to its actual size.

Direct Proportion

As one variable changes the other changes at the same rate.



4 cans of pop = £2.40

4 cans of pop = £2.40

2 cans of pop = £1.20

This multiplier is the same in the same way that this would be for ratio

This is a multiplicative change

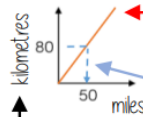
4 cans of pop = £2.40

12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph. Using a ruler helps for accuracy. Showing your conversion lines help as a "check" for solutions

Conversion between currencies



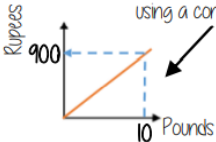
£1 = 90 Rupees

Currency is directly proportional

For every £1 I have 90 Rupees

£1 = 90 Rupees
 £10 = 900 Rupees

Currency can be converted using a conversion graph



Convert 630 Rupees into Pounds

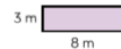
£1 = 90 Rupees
 £7 = 630 Rupees

Ratio between similar shapes



Angles in similar shapes do not change e.g. if a triangle gets bigger the angles can not go above 180°

The two rectangles are similar.



Corresponding sides

3m : 4.5m
 1m : 1.5m

8m : 12m
 1m : 1.5m

Note: Simplify to the same ratio

Understand Scale Factor

The two rectangles are similar.



$3 \times 15 = 45$

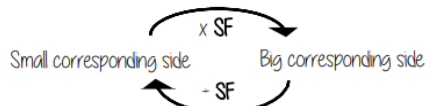
This is a multiplicative change.

Use corresponding sides to calculate a scale factor

Missing length
 $8 \times 15 = 12m$

Scale factor can also be calculated by:

Bigger corresponding side
 Smaller corresponding side



Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

The car image is 10cm

Image: Real life
 10cm : 300cm

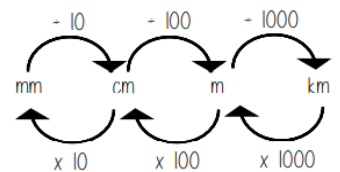


The car in real life is 210cm

Image: Real life
 7cm : 210cm



Interpret maps with scale factors



1 cm : 250 m

Ratios need to be in the same units

1 cm : 250m

1 cm : 25000cm

$250 \times 100 = 25000$

For every 1cm on my map is 25000cm in real life.



What do I need to be able to do?

By the end of this unit you should be able to:

- Form Expressions
- Expand and factorise single brackets
- Form and solve equations
- Solve equations with brackets
- Represent inequalities
- Form and solve inequalities

Keywords

Simplify: grouping and combining similar terms

Substitute: replace a variable with a numerical value

Equivalent: something of equal value

Coefficient: a number used to multiply a variable

Product: multiply terms

Highest Common Factor (HCF): the biggest factor (or number that multiplies to give a term)

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Form expressions

For unknown variables, a letter is normally used in its place

More than – **ADD**

Less than/ difference – **SUBTRACT**

e.g 4 more than t \longrightarrow $t + 4$
 8 less than k \longrightarrow $k - 8$

Only similar terms can be grouped together

e.g Find the perimeter of this shape
 (Perimeter = length around outside of shape)

$t + 2t + 1 + t + 2t + 1 \longrightarrow 6t + 2$

Directed numbers

$++ \longrightarrow +$

$-- \longrightarrow +$

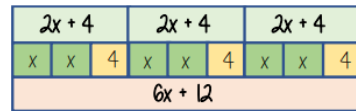
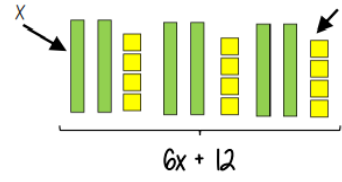
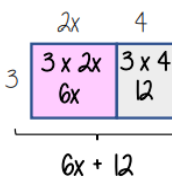
$+ - \longrightarrow -$

$- + \longrightarrow -$

e.g $a = -5$ and $b = 2$
 $a^2 = a \times a = -5 \times -5 = 25$
 $b + a = 2 + -5 = -3$

Multiply single brackets

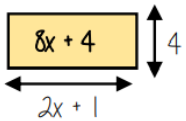
$3(2x + 4)$



Different representations of $3(2x+4) = 6x + 12$

Factorise into a single bracket

$8x + 4$



Try and make this the highest common factor

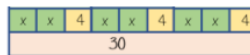
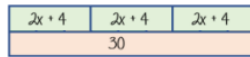
The two values **multiply** together (also the area) of the rectangle

$8x + 4 \equiv 4(2x + 1)$

Note:
 $8x + 4 \equiv 2(4x + 2)$
 This is factorised but the HCF has not been used

Solve equations with brackets

$3(2x + 4) = 30$



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \quad -12$

$6x = 18$

$-6 \quad -6$

Substitute to check your answer
 This could be negative or a fraction or decimal

$\frac{x}{3} \quad x = 3$

Simple Inequalities

$<$ less than

\leq Less than or equal to

$>$ More than

\geq More than or equal to

$x < 10$
 Say this out loud
 "x is a value less than 10"

$10 > x$
 Say this out loud
 "10 is more than the value"

Note:
 $x < 10$ and $10 > x$
 represent the same values

$x + 2 \leq 20$
 "my value + 2 is less than or equal to 20"
 $x \leq 18$
 The biggest the value can be is 18

Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

Form

$x \longrightarrow x3 \longrightarrow +2 \longrightarrow 11$
 $3x + 2 > 11$

Solve

$x \longleftarrow -3 \longleftarrow -2 \longleftarrow 11$
 $x > 3$

Check

This would suggest any value bigger than 3 satisfies the statement
 $3 \times 3 + 2 = 11 \checkmark \quad 10 \times 3 + 2 = 32 \checkmark$

Algebraic constructs

Expression

A sentence with a minimum of two numbers and one maths operation

Equation

A statement that two things are equal

Term

A single number or variable

Identity

An equation where both sides have variables that cause the same answer includes \equiv

Formula

A rule written with all mathematical symbols e.g area of a rectangle $a = b \times h$